Properties of Quadratic Functions

Because the vertex is the highest or lowest point on a parabola, its y-coordinate is the maximum value or minimum value of the function. The vertex of a parabola lies on the axis of the parabola. So, the graph of the function is increasing on one side of the axis and decreasing on the other side.

**EXAMPLE**  Analyzing a Quadratic Function

Describe the domain and range of \( f(x) = -\frac{1}{2}x^2 + 4x - 1 \). Then determine where the function is increasing and decreasing.

**SOLUTION**

From the original function, it follows that \( a = -\frac{1}{2}, b = 4, \) and \( c = -1 \). Because \( a \) is negative, the parabola opens downward and the function has a maximum value. Calculate the coordinates of the vertex.

\[
x = -\frac{b}{2a} = -\frac{4}{2\left(-\frac{1}{2}\right)} = 4 \quad f(4) = -\frac{1}{2}(4)^2 + 4(4) - 1 = 7
\]

The vertex of the parabola is \((4, 7)\). So, the domain is all real numbers and the range is \( y \leq 7 \). The function is increasing to the left of \( x = 4 \) and decreasing to the right of \( x = 4 \), as shown in the figure.

**Exercises Within Reach**

**Analyzing a Quadratic Function**  In Exercises 1–6, describe the domain and range of the function, and determine where the function is increasing or decreasing.

1. \( f(x) = 4x^2 + 3 \)
2. \( g(x) = -2x^2 - 1 \)
3. \( h(x) = x^2 + 6x + 5 \)
4. \( y = -\frac{3}{2}x^2 + 6x \)
5. \( y = 3x^2 - 3x + 4 \)
6. \( y = -x^2 - 10x - 3 \)