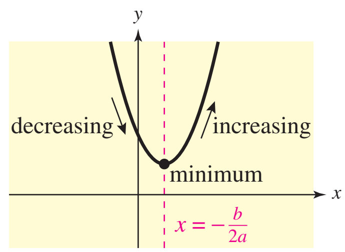
Properties of Quadratic Functions

Because the vertex is the highest or lowest point on a parabola, its y-coordinate is the *maximum value* or *minimum value* of the function. The vertex of a parabola lies on the axis of the parabola. So, the graph of the function is *increasing* on one side of the axis and *decreasing* on the other side.

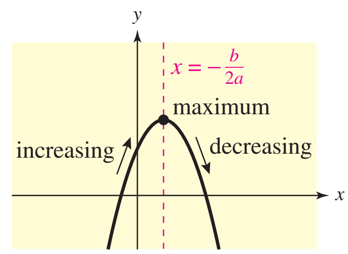
Properties of Quadratic Functions

$f(x) = ax^2 + bx + c,\ a > 0$



- Domain: All real numbers
- Range: $y \geq f\left(-\frac{b}{2a}\right)$
- Decreasing to the left of $x = -\frac{b}{2a}$
- Increasing to the right of $x = -\frac{b}{2a}$

$f(x) = ax^2 + bx + c,\ a < 0$



- Domain: All real numbers
- Range: $y \leq f\left(-\frac{b}{2a}\right)$
- Increasing to the left of $x = -\frac{b}{2a}$
- Decreasing to the right of $x = -\frac{b}{2a}$

EXAMPLE **Analyzing a Quadratic Function**

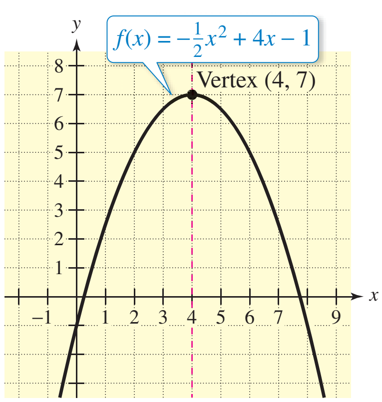
Describe the domain and range of $f(x) = -\frac{1}{2}x^2 + 4x - 1$. Then determine where the function is increasing and decreasing.

SOLUTION

From the original function, it follows that $a = -\frac{1}{2}$, $b = 4$, and $c = -1$. Because a is negative, the parabola opens downward and the function has a maximum value. Calculate the coordinates of the vertex.

$$x = -\frac{b}{2a} = -\frac{4}{2\left(-\frac{1}{2}\right)} = 4 \qquad f(4) = -\frac{1}{2}(4)^2 + 4(4) - 1 = 7$$

The vertex of the parabola is (4, 7). So, the domain is all real numbers and the range is $y \leq 7$. The function is increasing to the left of $x = 4$ and decreasing to the right of $x = 4$, as shown in the figure.



Exercises Within Reach®

Analyzing a Quadratic Function **In Exercises 1–6, describe the domain and range of the function, and determine where the function is increasing or decreasing.**

1. $f(x) = 4x^2 + 3$
2. $g(x) = -2x^2 - 1$
3. $h(x) = x^2 + 6x + 5$
4. $y = -\frac{3}{2}x^2 + 6x$
5. $y = 3x^2 - 3x + 4$
6. $y = -x^2 - 10x - 3$